CONFERENCE PHENOMENA IN HIGH DIMENSION
IHP, PARIS, 7-10 JUNE, 2022
TITLES AND ABSTRACTS

R. Adamczak, University of Warsaw, Poland
Random G-circulant matrices. Spectrum of random convolution operators on large finite groups
Abstract: I will describe the asymptotic behaviour of spectral measures of random G-circulant matrices, i.e., matrices corresponding to convolutions with random functions on a finite (not necessarily Abelian) group, when the order of the group tends to infinity. If time permits, I will also mention asymptotic freeness for collections of independent random matrices and central limit theorems for linear eigenvalue statistics. I will conclude with some open problems.

S. Artstein-Avidan, Tel-Aviv University, Israel
Dualities everywhere
Abstract: We will discuss and characterize order reversing quasi involutions and the multitude of dualites they induce, as well as the new insights these give regarding transforms well studied in convexity and in optimal transport.

F. Barthe, Université de Toulouse, France
A Gaussian correlation inequality for plurisubharmonic functions
Abstract: A positive correlation inequality is established for circular-invariant plurisubharmonic functions, with respect to complex Gaussian measures. It extends the complex Gaussian moment inequality of Arias de Reyna. The proof involves the Ornstein-Uhlenbeck semigroup, and another natural semigroup associated to the Gaussian Kähler Laplacian. Joint work with Dario Cordero-Erausquin.

A. Ben-Hamou, Sorbonne, France
Cutoff for permuted Markov chains
Abstract: For a given finite Markov chain with uniform stationary distribution, and a given permutation on the state-space, we consider the Markov chain which alternates between random jumps according to the initial chain, and deterministic jumps according to the permutation. In this framework, Chatterjee and Diaconis (2020) showed that when the permutation satisfies some expansion condition with respect to the chain, then the mixing time is logarithmic, and that this expansion condition is satisfied by almost all permutations. We will see that the mixing time can even be characterized much more precisely: for almost all permutations, the permuted chain has cutoff, at a time which only depends on the entropic rate of the initial chain.

S. Bobkov, University of Minnesota, USA
Maximum and decay of convolved densities
Abstract: First I will review several results on upper bounds for the maximum of probability densities on the Euclidean space that are representable as convolutions. In the second part, I will be discussing some new observations on how one can bound these densities pointwise by

means of the Legendre transform, with applications to the central limit theorem in terms of Renyi divergence and to the Gaussian decay of Bernoulli convolutions.

C. Bordenave, CNRS, Institut de Mathématiques de Marseille, France

TBA

Abstract: TBA

B. Dadoun, New York University Abu Dhabi, UAE

Variance conjecture in Schatten balls

Abstract: We present high dimensional asymptotics for the Hilbert-Schmidt norm of a self-adjoint matrix uniformly distributed in p-Schatten balls. When $p > 3$, the asymptotic expansion allows us to establish a generalized version of the variance conjecture for the family of p-Schatten unit balls of self-adjoint matrices. Joint work with Matthieu Fradelizi, Olivier Guédon and Pierre-André Zitt.

A. Eskenazis, Trinity College, Cambridge, UK

Learning low-degree functions on the discrete hypercube

Abstract: Let f be an unknown function on the n-dimensional discrete hypercube. How many values of f do we need in order to approximately reconstruct the function? In this talk we shall discuss the random query model for this fundamental problem from computational learning theory. We will explain a newly discovered connection with a family of polynomial inequalities going back to Littlewood (1930) which will in turn allow us to derive sharper estimates for the query complexity of this model, exponentially improving those which follow from the classical Low-Degree Algorithm of Linial, Mansour and Nisan (1989). Time permitting, we will also show a matching information-theoretic lower bound. Based on joint works with Paata Ivanisvili (UC Irvine) and Lauritz Streck (Cambridge).

A. Guillin, University Blaise Pascal, Clermont-Ferrand, France

Mean field system with singular interactions

Abstract: When studying Gaussian random matrices in high dimension, one often focus on the behavior of its eigenvalues which can be shown to follow a stochastic differential equation with a singular mean field interaction term of Coulomb type. This SDE has been introduced by Dyson and only qualitative results are known on the limit of its empirical distribution (Rogers-Shi, Cepa-Lepingle, Li-Li-Xie). After introducing the general context of propagation of chaos phenomenon, the talk will focus on two different methods: a probabilistic one by coupling enabling to get the first uniform in time quantitative result for Dyson's case, and an analytic one for the vortex 2D model. Based on joint works with P. Le Bris and P. Monmarché.

B. Klartag, Weizmann Institute, Israel

Bourgain's slicing problem and KLS isoperimetry up to polylog

Abstract: We prove that Bourgain's hyperplane conjecture and the Kannan-Lovasz-Simonovits (KLS) isoperimetric conjecture hold true up to a factor that is polylogarithmic in the dimension. Joint work with J. Lehec.

R. Latała, University of Warsaw, Poland

Spectral norm of randomized circulant matrices

Abstract: We discuss two-sided bounds for the expected operator norm of random matrices with non-homogenous independent entries. We formulate a lower bound for Rademacher matrices and conjecture that it may be reversed up to a universal constant. We show that our conjecture holds up to $\log\log n$ factor for randomized $n \times n$ circulant matrices and that the double logarithm may be eliminated under some mild additional assumptions on the coefficients. Based on a joint work with Witold Świątkowski.

G. Livshyts, Georgia Tech, USA

On an inequality somewhat related to the Log-Brunn-Minkowski conjecture

Abstract: I shall state and sketch a neat inequality concerning convex bodies and semi-norms, and show that cylinders (i.e. direct products of intervals with convex bodies of lower dimension) give equality cases for it. I shall explain its connections to the Log-Brunn-Minkowski conjecture. If time permits, I will discuss related results and surrounding questions. Based on a joint work with A. V. Kolesnikov.

M. Madiman, University of Delaware, USA

Submodularity in convex geometry

Abstract: Sumset estimates, which provide bounds on the cardinality of sumsets of finite sets in a group, form an essential part of the toolkit of additive combinatorics. We study analogues of these sumset estimates in the context of convex geometry. First, we observe that, with respect to Minkowski summation, volume is supermodular to arbitrary order on the space of convex bodies. Second, we explore sharp constants in the convex geometry analogues of the Plunnecke-Ruzsa inequalities. Third, we show the equivalence, in the class of zonoids, between the log-submodularity of volume, a local Alexandrov-Fenchel inequality, a local Loomis-Whitney inequality, and the Dembo-Cover-Thomas conjecture on the monotonicity of the ratio of volume to the surface area. We conjecture that all these inequalities are true for zonoids in arbitrary dimensions, and verify this conjecture in dimensions 2 and 3. Along the way, we give a negative answer to a question of Adam Marcus regarding the roots of the Steiner polynomial of zonoids. The talk is based on joint work with Matthieu Fradelizi, Mathieu Meyer, and Artem Zvavitch.

J. Najim, CNRS, LIGM, Université Gustave Eiffel, France

Properties of large Lotka-Volterra systems of coupled differential equations with random interactions

Abstract: Lotka-Volterra (LV) systems of coupled differential equations are widely used in mathematical biology and theoretical ecology to model populations with interactions. In the case of a large system, the random interactions form a large random matrix the statistical properties of which reflect (to some extent) some properties of the underlying biological system. In this talk, we will present some properties of such large LV systems (existence of a unique equilibrium, its positivity, and its stability properties) for some standard random matrix models (circular, elliptic, sparse). Based on joint works with Akjouj, Clenet, El Ferchichi, Massol.

P. Nayar, University of Warsaw, Poland

Slicing ℓ_p-balls: reload

Abstract: We will use a formula for volume of sections of convex bodies via negative moments to prove optimal stability results for hyperplane sections of balls in ℓ_p norms with finite p.

G. Paouris, Texas A.& M. University, USA

Isoperimetric inequalities for subspaces of L^p

Abstract: We establish a family of isoperimetric inequalities for polar L^p-centroid bodies in n dimensions when p is on the $[-1, 1]$ interval. We also establish empirical versions of these inequalities. Based on joint work in progress with R. Adamczak, P. Pivovarov and P. Simanjuntak.

G. Peccati, University of Luxembourg, Luxembourg

Variations on a theorem by de Jong

Abstract: In a classical contribution from 1990, P. de Jong established the surprising fact that an arbitrary sequence of normalized and degenerate U-statistics converges in distribution towards a Gaussian random variable if and only if its fourth cumulant converges to zero (and a certain Lindberg-type condition is satisfied). In this talk, I will describe some recent extensions of this result to the multi-dimensional and functional settings. In particular, our findings yield functional versions of the "universality of Wiener chaos phenomenon" first detected in Nourdin, Peccati, and Reinert (2010).
My talk is based on the following joint works:
- Ch. Döbler, M. Kasprzak and G. Peccati: Weak convergence of U-processes with size-dependent kernels. Ann. App. Prob., 2022
- Ch. Döbler, M. Kasprzak and G. Peccati. The multivariate functional de Jong CLT. Probab. Th. Rel. Fields, 2022+

M. Rudelson, University of Michigan, USA

When a system of real quadratic equations has a solution

Abstract: The existence and the number of solutions of a system of polynomial equations in n variables over an algebraically closed field is a classical topic in algebraic geometry. Much less is known about the existence of solutions of a system of polynomial equations over reals. Any such problem can be reduced to a system of quadratic equations by introducing auxiliary variables. Due to the generality of the problem, a computationally efficient algorithm for determining whether a real solution of a system of quadratic equations exists is believed to be impossible. We will discuss a simple and efficient sufficient condition for the existence of a solution. While the problem and the condition are of algebraic nature, the proof relies on Fourier analysis and concentration of measure.

P.-M. Samson, Université Gustave Eiffel, France

Criteria for entropic curvature on graphs along Schrödinger bridges at zero temperature

Abstract: According to the Lott–Sturm–Villani theory of entropic curvature, the displacement convexity property of entropy along W_2-Wasserstein geodesics allows to extend to geodesics spaces Bakry-Emery curvature conditions on Riemannian Manifold. This convexity property has been therefore used as a guideline to define curvature notions on graphs structures by many authors. In this talk, we focus on the use of displacement interpolations of probability distributions on discrete metric graphs du to C. Léonard that we call Schrödinger bridges at zero temperature. As a remarkable fact, these Schrödinger bridges are W_1-Wasserstein geodesics (according to the graph distance) supported by discrete geodesics of the space. We will present geometric conditions on the balls of the graph of radius 2 to obtain convexity properties of entropy along this Schrödinger bridges. As a byproduct, one gets new types of Prékopa-Leindler inequalities in discrete spaces, new transport-entropy inequalities related to refined concentration properties, and also modified logarithmic Sobolev inequalities. These geometric conditions can be interpreted as local curvature bounds, easy to estimate for many regular graphs. For example on the discrete hypercube, our strategy provides a new $W_2 - W_1$ Transport-entropy inequality that implies the well-known Talagrand's transport inequality for the standard Gaussian measure.

Y. Shenfeld, Princeton University, USA

Renormalization groups, transport maps, and multiscale Bakry-Émery criteria

Abstract: One of the important theoretical tools of the last 70 years in quantum and statistical field theories is the renormalization group in its various incarnations. In this talk, I will provide a new perspective on the subject by showing how Polchinski's exact version of the renormalization group gives rise, via the multiscale Bakry-Émery criterion, to Lipschitz transport maps between Gaussian free fields and quantum and statistical field theories. Consequently, many functional inequalities can be verified for the latter field theories, going beyond the current known results.

K. Tatarko, University of Alberta, Canada

L_p Steiner formula and its coefficients

Abstract: In this talk, we explore the L_p Steiner formula for the L_p affine surface area. We introduce the coefficients that arise in this formula that we call L_p-Steiner quermassintegrals and discuss their properties. It turns out that they possess some nice properties. In particular, they are new valuations on the set of convex bodies. Based on joint works with E. Werner.

T. Tkocz, Carnegie Mellon University, USA

Slicing ℓ_p-balls: load

Abstract: We will present general formulae for volume of sections of convex bodies via negative moments and show some applications, mainly nearly optimal stability results for hyperplane sections of the cube.

R. Vershynin, University of California, USA

Private measures, random walks, and synthetic data

Abstract: Differential privacy, a concept in theoretical computer science, challenges probabilists to handle exponentially rare events. In this talk, we consider the problem of creating differentially private synthetic data. More generally, we ask how to make any probability measure on a metric space differentially private. A new mechanism toward solving this problem is a superregular random walk, which is globally almost bounded (unlike Brownian motion), but locally is statistically indistinguishable from a random walk with independent increments. This is joint work with March Boedihardjo and Thomas Strohmer.

E. Werner, Case Western Reserve University, USA

Spherical convex hull of random points on a wedge

Abstract: Consider two half-spaces H_1^+ and H_2^+ in $\mathbb{R}^{d+1}$ whose bounding hyperplanes H_1 and H_2 are orthogonal and pass through the origin. The intersection $\mathbb{S}^d_{2,+} := \mathbb{S}^d \cap H_1^+ \cap H_2^+$ is a spherical convex subset of the d-dimensional unit sphere $\mathbb{S}^d$ and is called a spherical wedge. Choose n independent random points uniformly at random on $\mathbb{S}^d_{2,+}$ and consider the expected facet number of the spherical convex hull of these points. It is shown that, up to terms of lower order, this expectation grows like a constant multiple of $\log n$. The result is compared to the corresponding behavior of classical Euclidean random polytopes and of spherical random polytopes on a half-sphere. Based on joint work with Florian Besau, Anna Gusakova, Matthias Reitzner, Carsten Schuett and Christoph Thaele.

P. Youssef, New York University Abu Dhabi, UAE

Monotonicity of entropy for random matrices

Abstract: In a celebrated result of Artstein-Ball-Barthe-Naor, it has been shown that the classical entropy is monotone along the central limit theorem. We explore this concept in the context of random matrix theory, investigating monotonicity properties of entropy along Wigner semi-circle theorem. This leads us to consider the same in the limiting regime of random matrices which is governed by Free Probability. This is a joint work with Benjamin Dadoun.